

Power Allocation in the Energy Harvesting Full-Duplex Gaussian Relay Channels

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Abstract—In this paper, we propose a general model to study the full-duplex non-coherent decode-and-forward Gaussian relay channel with energy harvesting (EH) nodes, called NC-EH-RC, in three cases: *i*) no energy transfer (ET), *ii*) one-way ET from the source (S) to the relay (R), and *iii*) two-way ET. We consider the problem of optimal power allocation in NC-EH-RC in order to maximize the total transmitted bits from S to the destination in a given time duration. General stochastic energy arrivals at S and R with known EH times and amounts are assumed. In NC-EH-RC with no ET, the complicated min-max optimization form along with its constraints make the problem intractable. It is shown that this problem can be transformed to a solvable convex optimization form; however, convex optimization solution does not provide the structural properties of the optimal solution. Therefore, following an alternative perspective, we investigate conditions on harvesting process of S and R where we find optimal algorithmic solution. Further, we propose some suboptimal algorithms and provide some examples, in which the algorithms are optimal. Moreover, we find a class of problems for NC-EH-RC with one-way ET from S to R, where the optimal algorithmic solution is devised. For NC-EH-RC with two-way ET, we propose general optimal algorithmic solution. Furthermore, the performance of the proposed algorithms are evaluated numerically and compared with optimal numerical convex optimization tools.

Index Terms—Convex optimization, energy harvesting, energy transfer, full-duplex, Gaussian relay channel, power allocation.

I. INTRODUCTION

RECENTLY, Energy Harvesting (EH) has received considerable research interest as a promising solution to the perennial energy constraint of wireless networks with limited batteries [1]. Moreover, in near future, increasing energy consumption of highly-demanded mobile data networks is anticipated to be the main cause of global warming. Hence, EH has emerged to be used as a foundation of green communication networks [2]. Energy harvesters collect ambient energy from the environment (including solar, hydro, wind, biomass, vibration, geothermal, piezoelectricity) and convert it into usable electrical energy. In contrast to the conventional battery-powered nodes, EH nodes have access to an unlimited source of energy which is free for users. However, the limitations in EH nodes are the low EH production rate as well as its sporadic nature. To overcome these limitations, sophisticated utilization of scavenged energy is mandatory.

Another related novel research avenue focuses on providing the power of devices wirelessly through ambient Radio Frequency (RF) signals. This avenue, known as wireless energy transfer, is motivated by notable development for the coupled magnetic resonators in [3] and has considerable increasingly emerging applications [4]–[6]. Also, recently authors in [7] designed an efficient rectenna, which is capable of harvesting ambient RF energy. Wireless energy transfer consists of two research directions: one direction considers *Simultaneous Wireless Information and Power Transfer* and characterizes the achievable rate-energy trade-off (see e.g., [8], [9] and the references therein). Another direction aims to design a new type of networks, called *Wireless Powered Communication Networks*, where the nodes harvest their required powers from wireless power transfer (see e.g., [6], [10], and the references therein).

A. Related Work and Motivation

EH has been considered as a facility to ameliorate the energy consumption challenge of sensor nodes in many pioneering works [11]–[13]. Information theoretic capacity of AWGN channels with an EH transmitter has been derived in [14]. In a similar work, [15] has derived the Shannon capacity of sensor nodes by considering processing energy cost, energy inefficiencies and channel fading. In [16], the authors have studied the optimal packet scheduling problem in wireless single-user EH communication system, where energy and data packets are stochastically arrived at the source node: to minimize the transmit time of the data packets, transmission rate adaptively changes according to data and energy traffics. This optimal packet scheduling has later been extended to fading channel [17], broadcast channel [18], [19], multiple-access channel [20], two-hop channel [21] and interference channel [22].

The wireless Relay Channel (RC) is a basic model to investigate the benefits of cooperation in communication networks from many aspects such as information theoretic capacity, diversity, outage analysis, cooperative and network coding, resource allocation, etc. In addition, resource-constrained networks such as Wireless Sensor Network (WSN) can get more benefit of cooperation through optimal allocation of energy and bandwidth to the nodes based on the available channel state information of those nodes (see e.g. [23] and the references therein). Motivated by the advantages that the EH and cooperation provide for the next generation wireless networks (such as high data rates, energy efficiency, and so on), a fundamental question is to find the optimal resource allocation in a RC with EH nodes.

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Some special cases of multi-hop and relay channel with EH transmitting nodes have been considered in [21], [24], [25]. The authors in [21] have considered an EH two-hop network with only the relay node harvesting the energy. In [24], authors have studied a two-hop network where both Source (S) and Relay (R) are the EH nodes. In [25], *half-duplex* orthogonal RC with Decode-and-Forward (DF) relay has been considered and two different delay constraints are investigated: one-block decoding delay constraint and arbitrary decoding delay constraint (up to total transmission blocks). On the other hand, Full-Duplex (FD) protocols has emerged recently to overcome the spectral efficiency loss of half-duplex protocols, by allowing the users to send and receive information concurrently at a same frequency band (see e.g., [26], [27] and the references therein). In [28], we have considered the general model for RC with a *direct link* and *FD coherent* DF relaying strategy. So a more complicated min-max optimization problem has arisen in [28] which has not been encountered in prior works. The complicated min-max problem was transformed to a solvable convex optimization form, using some mathematical background. First, an auxiliary parameter was introduced and then a minimax theorem of [29] was used to make the problem tractable. However, the convex optimization solutions derived in [28] for FD RC do not provide detailed structural properties of optimal transmission policy. In fact, general algorithmic solution for the FD coherent DF Gaussian RC has not been tackled in the previous literatures. Moreover, this problem is not easily reducible to other channels like point-to-point, multiple-access channel, broadcast channel, two-hop channel, etc. None of the aforementioned works have considered the energy transfer. In the context of wireless powered communication networks, the authors in [30] have introduced the notion of *energy cooperation* where users share a portion of their scavenged energy in order to shape and optimize the energy arrivals to improve the overall performance. Here, cooperation is performed in the battery energy level instead of signal level as in the classical cooperative networks.

B. Main Contributions and Organization

In this paper, we consider the problem of optimal power allocation for a three-node FD Gaussian RC with EH nodes. We focus on noncoherent DF relaying strategy compared to the coherent strategy in [28]. Although the noncoherent DF bound on the capacity of the RC is lower than that of the coherent DF lower bound (which is the capacity of the degraded RC); implementing noncoherent communication is more convenient in wireless systems. Our goal is to maximize the total number of bits that can be delivered from S node to the destination (D) node in a given time duration. Three cases are studied based on the ability of the nodes to transfer some parts of their harvested energy: (i) no Energy Transfer (ET) among nodes (ii) one-way ET from S to R, and (iii) two-way ET between S and R or bi-directional energy cooperation. We consider a general model compared to the existing works. In our model, there is a *direct link* from S to D (in contrast to [21], [24], [30]) and also we investigate the FD mode compared to the half-duplex mode of [25]. Besides, unlike [21], [24], [25], [28], we consider the

energy transfer among nodes, which makes our model more general. We assume zero cost energy transfer among nodes. Studying the cost of energy sharing among nodes is parallel to our work (see e.g., [30] and the references therein). In this work, we investigate the offline problem where we assume the availability of offline knowledge about EH times and amounts in S and R. This is due to the fact that the online problem that assigns the nodes' powers in real-time is intractable for now in our studied model and it is consistent with the assumptions in existing works, such as [16]–[22].

In our problem, like [28], the structural properties of optimal policy can not be derived from the convex optimization solution. Therefore, we follow a different perspective to derive the algorithmic solutions. Our main contributions in the rest of this paper are organized as follows.

- In Section II, we propose a general model for FD Non-Coherent DF Gaussian RC with EH nodes, called “NC-EH-RC” in three cases: 1) no-ET, 2) one-way ET from S to R, and 3) two-way ET. Also, relaying strategy and harvesting process are described and some preliminaries are added to make the paper self-contained.
- Section III considers NC-EH-RC with no ET case. First, we formulate the power allocation problem. Then, we show that it can be transformed to a tractable form even though it is a complicated optimization problem. However, the solution do not provide the detailed structural properties of optimal solution. Hence, we explore some conditions on the harvesting process of R that help us to find the optimal algorithmic solution. We provide this solution when the R is in good EH condition, which means that R can forward any received information from S toward D without any energy shortage. This solution reveals some important specifications of general optimal solution for our problem that discriminate it from other problems solved in the literatures. Moreover it is shown that disjoint optimization at S and R is suboptimal, though it is optimal in some special cases. We further propose a suboptimal algorithmic solution based on *total* power allocation for S and R, which is optimum for some realization of EH process at S and R.
- In Section IV, we study optimal power allocation in NC-EH-RC with one-way ET from S to R. For some conditions on the harvesting process of S, we propose an algorithmic optimal solution for our power allocation problem. We devise an algorithm for optimal power allocation, when the S is in good EH condition. It means that for a fixed amount of network's energy resources, S transfers some parts of its harvested energy (stored in its battery) to R in order to improve the performance.
- In Section V, we concentrate on the optimal power allocation for NC-EH-RC with two-way ET or bi-directional energy cooperation. We propose a *general* algorithmic optimal solution for the problem in this case. In fact, two-way ET capability provides new interesting specifications for optimal solution. These are utilized for devising algorithms that solve the problem optimally.
- In Section VI, we evaluate the performance of our pro-

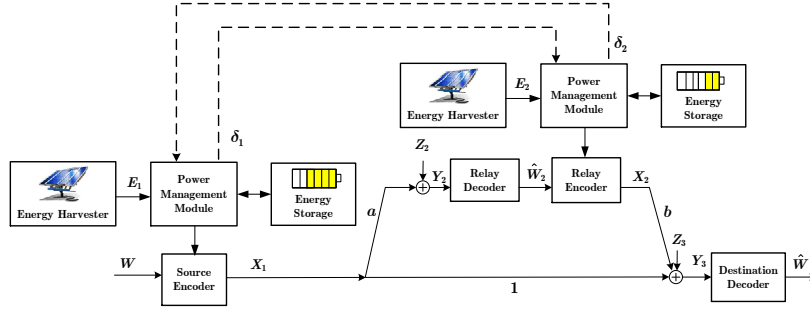


Fig. 1. General model for Gaussian RC with EH nodes that are capable of transferring energy to each other.

posed algorithmic solutions derived in sections III, IV, and V, numerically. Also, we present some typical examples, where each of the suboptimal solutions outperforms the other one and is optimal. Besides, allocated powers of nodes in our algorithms are compared with optimal numerical convex optimization tool.

- Finally, section VII concludes the paper.

II. SYSTEM MODEL AND PRELIMINARIES

Notation: Upper-case letters (e.g., X) denote Random Variables (RVs) and lower-case letters (e.g., x) their realizations. The probability mass function (p.m.f) of a RV X with alphabet set \mathcal{X} is denoted by $p_X(x)$. The variables related to S and R are indicated with subscripts 1 and 2, respectively. We show N -length vectors by bold-face letters, (e.g., $\mathbf{V}_i = [V_i^1, V_i^2, \dots, V_i^N]$), where their j -th component is denoted by superscript j (e.g., V_i^j). $\mathcal{CN}(0, \sigma^2)$ denotes a zero-mean complex value Gaussian distribution with variance σ^2 and $I(X; Y)$ denotes the *mutual information* between X and Y . In addition, $\mathcal{C}(x) = \log_2(1 + x)$ and $[x]^\dagger = \max\{1, x\}$.

RC models a three-node network, in which the source node wants to communicate to the destination node with the help of the relay node. Codebook, encoder, decoder and rate for the discrete memoryless RC (DM-RC) can be defined as [31, Chapter 16].

A general model for Gaussian RC with EH nodes and ET capabilities is depicted in Fig. 1. The channel outputs corresponding to the channel inputs X_1, X_2 are as follows

$$Y_2 = aX_1 + Z_2, \quad (1)$$

$$Y_3 = X_1 + bX_2 + Z_3, \quad (2)$$

where a and b are channel gains of S-R and R-D links, respectively, assuming normalized channel gain for the S-D link, and we have $Z_2 \sim \mathcal{CN}(0, N_0)$, $Z_3 \sim \mathcal{CN}(0, N_0)$.

The energy harvester block scavenges the ambient energy from the environment. Transmitting nodes are capable of transferring parts of their harvested energies to each other to have better control on the network's energy resources. We study three cases in this paper, namely (i) nodes with no ET ($\delta_1 = \delta_2 = 0$ in Fig. 1), (ii) nodes with one-way ET from S to R ($\delta_1 \neq 0, \delta_2 = 0$ in Fig. 1), and (iii) nodes with two-way ET ($\delta_1 \neq 0, \delta_2 \neq 0$ in Fig. 1), where δ_1 and δ_2 denote the amount of energy transferred in $S \rightarrow R$ and $R \rightarrow S$ directions, respectively. The power management module determines the

amount of the harvested energy used for communication as well as the amount transferred to other node. The remaining energy is stored in the energy storage device (e.g., battery or super-capacitor) for future use.

A. Relaying Strategy

Since the capacity of FD DM-RC is not known in general, in this paper, we consider an achievable rate for the RC that provides a lower bound on its capacity. This rate is achieved by *noncoherent* DF strategy in the R. Since implementing of the coherent communication is difficult in wireless systems [31], it is more convenient to consider noncoherent coding schemes in S and R at the cost of losing some rate. In this strategy, R plays a central role in communication by decoding the information bits received from S. Then, S and R noncoherently cooperate to transmit their codewords to D. D decodes the received coded information from S and R, simultaneously. With this coding scheme, the following rate for DM-RC is achievable [31, Chapter 16]:

$$C \geq \max_{p_{X_1}(x_1)p_{X_2}(x_2)} \min \{I(X_1, X_2; Y_3), I(X_1; Y_2|X_2)\}. \quad (3)$$

Note that X_1 and X_2 are independent in this case. Hence, we used $p_{X_1, X_2}(x_1, x_2) = p_{X_1}(x_1)p_{X_2}(x_2)$. The corresponding rate for the Gaussian RC is given by

$$C \geq \min \left\{ \mathcal{C} \left(\frac{P_1 + b^2 P_2}{N_0} \right), \mathcal{C} \left(\frac{\max\{1, a^2\} P_1}{N_0} \right) \right\} \\ = \begin{cases} \mathcal{C} \left(\frac{P_1 + b^2 P_2}{N_0} \right) & \text{if } \frac{([a^2]^\dagger - 1)P_1}{b^2 P_2} > 1, \\ \mathcal{C} \left(\frac{[a^2]^\dagger P_1}{N_0} \right), & \text{otherwise,} \end{cases} \quad (4)$$

where P_1 and P_2 are the powers of S and R, respectively. The first term under the minimum can be interpreted as *Noncooperative Multiple-Access* (N-MAC) term. The second term implies that if the quality of S-R link is worse than that of the direct S-D link (i.e., $a^2 < 1$), any information that R can decode, is previously decoded at D. In such cases, R cannot help and should be ignored (The minimum is $\mathcal{C} \left(\frac{P_1}{N_0} \right)$, which is the capacity of direct link from S to D). We do not consider these cases in this paper.

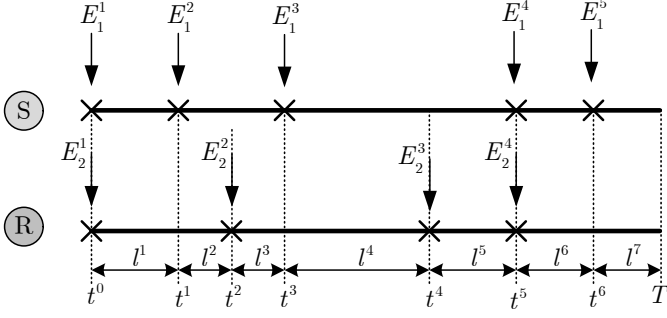


Fig. 2. EH instants and amounts for S and R with $K = 6$.

Remark 1: In two-hop networks, studied in [24], [32], there is a *data causality constraint* at R: data bits can only be transmitted from R toward D after they have arrived from S. With this constraint, in a given time duration, the minimum of total bits transmitted from S to R (called $\mathcal{B}_{S \rightarrow R}$) and the total transmitted bits from R to D (called $\mathcal{B}_{R \rightarrow D}$) is equal to $\mathcal{B}_{R \rightarrow D}$. Therefore, the min-max problem has not been encountered in these works. This again shows the complexity of our problem compared to that of [24], [32].

B. Harvesting Process

We consider a RC, in which the harvested energy from the environment is the sole source of energy in the network. Our problem is to maximize the number of bits delivered by a deadline T from S to D. S and R harvest energy at random instants $t^0, t^1, t^2, \dots, t^K$ and in random amounts $E_1^1, E_1^2, \dots, E_1^{K+1}$ and $E_2^1, E_2^2, \dots, E_2^{K+1}$, respectively. If at some instants only S or R harvests energy, we simply set the amounts of the energy harvested by the other one to zero (see Fig. 2). The interval between two harvesting instants is called an epoch. The length of i^{th} epoch is $l^i = t^i - t^{i-1}$ for $i = 1, \dots, K + 1$. So, there are a total of $K + 1$ epoch with $t^0 = 0$ and $t^{K+1} = T - t^K$. We consider the offline problem in which the $t^i, E_1^i, E_2^i, \forall i$ are known to S and R before the start of transmission. Moreover, considering the offline problem, enables transmitting nodes to share their harvested energies in the offline mode. We ignore any inefficiency in ET among nodes.

C. Optimal Point-to-Point Solution and Its Interpretation

To make the paper self-contained, we briefly describe the optimal packet scheduling in wireless point-to-point EH communication systems which was proposed in [16], translating to our notations. The problem in [16] is to find the optimal offline transmission policy, which minimizes the transmission completion time with a predefined data bits to transmit. Energies harvested at time instants t^i with the amount of E^i at the transmitter. This problem is shown to be the dual of maximizing the throughput (total number of bits that can be transmitted) in a given time (deadline), with the same optimal transmission policies [17]. The solution for these problems is as follows [16, Theorem 1]:

$$i_n = \underset{i: t^{i_n-1} < t^i \leq T}{\operatorname{argmin}} \frac{\sum_{j=i_n-1}^{i-1} E^j}{t^i - t^{i_n-1}}, \quad n = 1, \dots, N \quad (5)$$

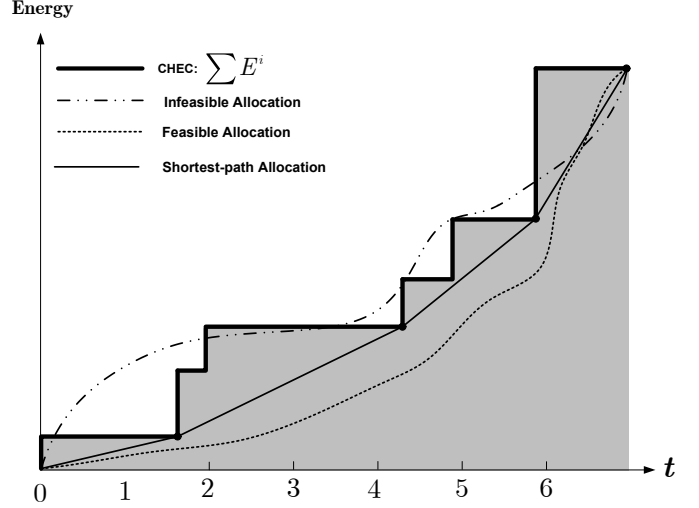


Fig. 3. Shortest path power allocation for point-to-point EH communication system. A typical feasible and infeasible solutions are included in this figure for a typical cumulative energy harvested curve (CEHC), shown in thick solid line.

$$P^n = \frac{\sum_{j=i_n-1}^{i_n-1} E^j}{t^{i_n} - t^{i_n-1}}, \quad n = 1, \dots, N \quad (6)$$

$$L^n = t^{i_n} - t^{i_n-1} = \sum_{j=i_{n-1}+1}^{i_n} l^j, \quad n = 1, \dots, N \quad (7)$$

where $P^i, \forall i$ is the sequence of transmission power with corresponding duration of $L^i, \forall i$, respectively, $T = \sum_{n=1}^N L^n$ and i_n is the index of time instant that allocated power changes from P^n to P^{n+1} . This result is achieved using some lemmas about necessary optimality conditions. The idea follows from the lazy scheduling [33], convexity [34] and majorization theory [35]. The solution has the *shortest-path* graphical interpretation. This notion is depicted in Fig. 3: any power allocation restricted to the Cumulative Energy Harvested Curve (CEHC), $\sum E^i$, from below is feasible; It is infeasible otherwise. The optimal solution is shown to be the piecewise linear curve with the shortest length (or tightest string) restricted to CEHC from below, connecting the origin to the end point of CEHC (shown by thin solid line in Fig. 3).

In next three sections, we investigate the optimal algorithmic power allocation solution in our NC-EH-RC model with no ET, one-way ET and two-way ET.

III. NC-EH-RC WITH NO ENERGY TRANSFER

In this section, we consider the NC-EH-RC with no ET. We study the optimal power allocation for S and R in order to maximize the total transmitted bits from S to D, satisfying energy causality constraints at S and R. This means that energy cannot be utilized in S and R before it is harvested in the corresponding node. We consider the noncoherent relaying strategy, achieving the rate in (4). Therefore, we can formulate the problem as:

$$\begin{aligned} \mathcal{L}(\{P_1^i\}, \{P_2^i\}, \xi, \mu, \vartheta, \eta) = & \sum_{i=1}^{K+1} \left\{ \lambda^i \mathcal{C} \left(\frac{P_1^i + b^2 P_2^i}{N_0} \right) + (1 - \lambda^i) \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right\} l^i \\ & - \sum_{k=1}^{K+1} \xi_k \left(\sum_{i=1}^k P_1^i l^i - \sum_{i=0}^{k-1} E_1^i \right) - \sum_{k=1}^{K+1} \mu_k \left(\sum_{i=1}^k P_2^i l^i - \sum_{i=0}^{k-1} E_2^i \right) + \sum_{i=1}^{K+1} \vartheta_i P_1^i + \sum_{i=1}^{K+1} \eta_i P_2^i, \end{aligned} \quad (25)$$

$$\max_{P_1, P_2} \sum_{i=1}^{K+1} \min \left\{ \tilde{C}_1(P_1^i, P_2^i), \tilde{C}_2(P_1^i) \right\} l^i \quad (8)$$

$$s.t. \quad P_1^i \geq 0, P_2^i \geq 0, \quad i = 1, \dots, K+1, \quad (9)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} E_1^i, \quad k = 1, \dots, K+1, \quad (10)$$

$$\sum_{i=1}^k P_2^i l^i \leq \sum_{i=0}^{k-1} E_2^i, \quad k = 1, \dots, K+1. \quad (11)$$

where we have $\tilde{C}_1(P_1^i, P_2^i) = \mathcal{C} \left(\frac{P_1^i + b^2 P_2^i}{N_0} \right)$ and $\tilde{C}_2(P_1^i) = \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right)$.

Equation (9) denotes the non-negativity of the powers at S and R. Equations (10) and (11) state the energy causalities at S and R, respectively. Finding the solution of the above problem is not straightforward as it has the min-max optimization form which cannot be separated due to the FD nature of the problem. In other words, since R sends and receives information at the same time, in each epoch we do not know which term (in epoch i , $\tilde{C}_1(P_1^i, P_2^i)$ or $\tilde{C}_2(P_1^i)$) is the minimum. Therefore, optimal power assignment for S and R to maximize the total transmitted bits is unknown. Also, observe that in (4) the condition that specifies the minimum term depends on the optimization parameters (i.e., \mathbf{P}_1 and \mathbf{P}_2 in (8)-(11)).

We rewrite the problem in (8)-(11) by introducing $0 \leq \lambda^i \leq 1$ as follows

$$\max_{\{P_1^i\}, \{P_2^i\}} \sum_{i=1}^{K+1} \min_{\{\lambda^i\}} \left\{ \lambda^i \tilde{C}_1(P_1^i, P_2^i) + (1 - \lambda^i) \tilde{C}_2(P_1^i) \right\} l^i \quad (12)$$

$$s.t. \quad P_1^i \geq 0, P_2^i \geq 0, \quad i = 1, \dots, K+1, \quad (13)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} E_1^i, \quad k = 1, \dots, K+1, \quad (14)$$

$$\sum_{i=1}^k P_2^i l^i \leq \sum_{i=0}^{k-1} E_2^i, \quad k = 1, \dots, K+1, \quad (15)$$

where,

$$\lambda^i = \begin{cases} 0 & \text{if } \tilde{C}_1(P_1^i, P_2^i) > \tilde{C}_2(P_1^i), \\ 1 & \text{if } \tilde{C}_1(P_1^i, P_2^i) < \tilde{C}_2(P_1^i), \\ \text{arbitrary} & \text{if } \tilde{C}_1(P_1^i, P_2^i) = \tilde{C}_2(P_1^i). \end{cases} \quad (16)$$

Noting the similarity of the problem in (12) with the one in [28], we use the technique proposed in [28, Theorem 1]

and change the order of min and max operators in (12) by applying the min-max theorem of Tarkelsen [29].

Then, the problem in (12)-(15) can be decomposed into the following two problems.

$$\begin{aligned} \text{(Problem 1)} : f^*(\{\lambda^i\}) = & \max_{\{P_1^i\}, \{P_2^i\}} \sum_{i=1}^{K+1} \left\{ \lambda^i \mathcal{C} \left(\frac{P_1^i + b^2 P_2^i}{N_0} \right) \right. \\ & \left. + (1 - \lambda^i) \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right\} l^i \end{aligned} \quad (17)$$

$$s.t. \quad P_1^i \geq 0, P_2^i \geq 0, \quad i = 1, \dots, K+1, \quad (18)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} E_1^i, \quad k = 1, \dots, K+1, \quad (19)$$

$$\sum_{i=1}^k P_2^i l^i \leq \sum_{i=0}^{k-1} E_2^i, \quad k = 1, \dots, K+1. \quad (20)$$

$$\text{(Problem 2)} : \min_{\{\lambda^i\}} f^*(\{\lambda^i\}) \quad (21)$$

$$s.t. \quad 0 \leq \lambda^i \leq 1, \quad i = 1, \dots, K+1. \quad (22)$$

Problems 1 and 2 are convex optimization problems as their objective functions are concave and their constraints are affine; thus, they can be solved by efficient convex optimization methods [34].

For Problem 1, we write the Lagrangian function for any $\xi_k \geq 0$, $\mu_k \geq 0$, $\vartheta_k \geq 0$ and $\eta_k \geq 0$ as (25), in top of this page. KKT optimality conditions for (25) are

$$\frac{\lambda^i}{P_1^i + b^2 P_2^i + N_0} + \frac{(1 - \lambda^i)[a^2]^\dagger}{[a^2]^\dagger P_1^i + N_0} - \sum_{k=i}^K \xi_k + \vartheta_i = 0, \quad \forall i \quad (23)$$

$$\frac{b^2 \lambda^i}{P_1^i + b^2 P_2^i + N_0} - \sum_{k=i}^K \mu_k + \eta_i = 0, \quad \forall i \quad (24)$$

together with following complementary slackness conditions;

$$\xi_k \left(\sum_{i=1}^k P_1^i l^i - \sum_{i=0}^{k-1} E_1^i \right) = 0, \quad k = 1, \dots, K, \quad (26)$$

$$\mu_k \left(\sum_{i=1}^k P_2^i l^i - \sum_{i=0}^{k-1} E_2^i \right) = 0, \quad k = 1, \dots, K, \quad (27)$$

$$\sum_{i=1}^N \vartheta_i P_1^i = 0, \quad i = 1, \dots, K+1, \quad (28)$$

$$\sum_{i=1}^N \eta_i P_2^i = 0, \quad i = 1, \dots, K+1. \quad (29)$$

We find the optimal solution as

$$P_1^{i*} = \frac{(1 - \lambda^i)}{\frac{1}{b^2} \sum_{k=i}^K \mu_k + \sum_{k=i}^K \xi_k + \frac{\eta_i}{b^2} - \vartheta_i} - \frac{N_0}{[a^2]^\dagger}, \quad \forall i \quad (30)$$

$$P_2^{i*} = \frac{\lambda^i}{\sum_{k=i}^K \mu_k - \eta_i} - \frac{(1 - \lambda^i)}{b^2 \sum_{k=i}^K \xi_k - \sum_{k=i}^K \mu_k - b^2 \vartheta_i + \eta_i} + \frac{N_0}{[a^2]^\dagger b^2} - \frac{N_0}{b^2} \quad (31)$$

To be able to solve Problem 2, one has to find the P_1^{i*} and P_2^{i*} in terms of only λ^i . Then, it is enough to solve the following problem with respect to λ^i s.

$$\min_{\{\lambda^i\}} \sum_{i=1}^{K+1} \left(\lambda^i \mathcal{C} \left(\frac{P_1^{i*} + b^2 P_2^{i*}}{N_0} \right) + (1 - \lambda^i) \mathcal{C} \left(\frac{[a^2]^\dagger P_1^{i*}}{N_0} \right) \right) l^i \quad (32)$$

$$s.t. \quad 0 \leq \lambda^i \leq 1, \quad i = 1, \dots, K+1. \quad (33)$$

As the solutions provided in (30) and (31) do not give any explicit idea about the structural properties of optimal power assignment; therefore, it is not straightforward to find algorithmic solutions for \mathbf{P}_1^* and \mathbf{P}_2^* using these expressions. In fact, finding a general algorithmic solution for optimal power allocation for NC-EH-RC is a complex and non-trivial task that has not been tackled in the existing works, yet. Therefore, in the following we find the optimal algorithmic solution for NC-EH-RC with no ET in a special case to gain insight on the optimal solution.

A. Optimal Algorithmic Solution for NC-EH-RC with No ET

In this case, the R is in good EH condition and has scavenged sufficient energy such that it is able to forward any information bits received from S. Algorithm 1 gives the optimal solution in this case. Its optimality is shown in Lemma 1.

Algorithm 1 Optimal greedy power allocation algorithm for NC-EH-RC with no ET, when R is in good EH condition.

(1) *Single-user Power Allocation for S*

$$o_n = \underset{o_{n-1} < i \leq K+1}{\operatorname{argmin}} \frac{\sum_{j=o_{n-1}}^{i-1} E_1^j}{t^i - t^{o_{n-1}-1}}$$

$$P_1^{n*} = \frac{\sum_{j=o_{n-1}}^{o_n-1} E_1^j}{t^{o_n} - t^{o_{n-1}-1}}$$

(2) *Feasibility Problem for Power Allocation at R*

if $P_2^i = \frac{[a^2]^\dagger - 1}{b^2} P_1^{i*}, \forall i$ **is feasible** **then**

Find power allocation for R as $P_2^{i*} = \frac{[a^2]^\dagger - 1}{b^2} P_1^{i*}, \forall i$

else

return The algorithmic optimal solution is not known in general.

end if

Definition 1: We define the class of problem (4) with R in good EH condition as the problems, where $P_1^{i*}, P_2^{i*}, \forall i$ (in optimal solution) satisfy

$$P_2^{i*} \geq \frac{[a^2]^\dagger - 1}{b^2} P_1^{i*}, \forall i.$$

This means that R harvests sufficient energy so that at any time instant, it has enough power to transfer any bits received from S toward D. The algorithm is called *greedy* when R uses the least power to transmit the data bits received by S, i.e., $P_2^{i*} = \frac{[a^2]^\dagger - 1}{b^2} P_1^{i*}, \forall i$.

Lemma 1: In NC-EH-RC with no ET, Algorithm 1 provides the optimal greedy power allocation, when R is in good EH condition.

Proof: If R is in good EH condition, the bottleneck is the S-R link. Hence, the cost function is expressed as the second term under the minimum and the problem is

$$\max_{P_1, P_2} \sum_{i=1}^{K+1} \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) l^i$$

$$s.t. \quad (9) - (11)$$

Removing variables and constraints irrelevant to the cost function results in

$$\max_{P_1} \sum_{i=1}^{K+1} \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) l^i$$

$$s.t. \quad P_1^i \geq 0, \forall i \quad \text{and (10)}$$

Now, the cost function is a concave function of its single variable \mathbf{P}_1 and the constraints are convex sets over \mathbf{P}_1 . Hence, the problem is convex and the solution is the shortest-path power allocation algorithm for the S. Now, R should only use sufficient power to make $\tilde{C}_1(P_1^i, P_2^i) \geq \tilde{C}_2(P_1^i), \forall i$ or $\mathcal{C} \left(\frac{P_1^i + b^2 P_2^i}{N_0} \right) \geq \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right), \forall i$. This is equivalent to satisfy $P_1^i + b^2 P_2^i \geq [a^2]^\dagger P_1^i, \forall i$, as $\mathcal{C}(\cdot)$ is a monotonic function. Algorithm 1 utilizes the least possible power for R in each epoch to achieve the above result, so it is called the *greedy* algorithm. ■

Remark 2: In existing works, the EH nodes must use all their harvested energy in order to be optimal [16]–[21], [24], [32]. However, Lemma 1 shows that for NC-EH-RC, leaving some parts of harvested energy unused in the battery of R is not necessarily suboptimal. This fact shows that our problem can not be reduced to the existing EH problems in [16]–[21], [24], [32]. In addition, if we do not use a greedy algorithm, any feasible power allocation for R, greater than the one in Algorithm 1, also provides the optimal solution for the problem. Therefore, Lemma 1 reveals a general specification of the solution of our general problem in (8)–(11) in the following lemma.

Lemma 2: The optimal power allocation for NC-EH-RC is not necessarily unique.

Outline of proof: Consider an optimal power allocation that in each epoch the second term under the minimum of (4) is dominant (the achievable rate of channel is forced by the rate of S-R channel). In such cases, it is obvious that expending more power by the R, not violating its energy causality constraint, has no benefit in terms of total transmitted bits in a given deadline and leads to the same optimal value. Therefore, \mathbf{P}_1^* and \mathbf{P}_2^* are not necessarily unique. ■

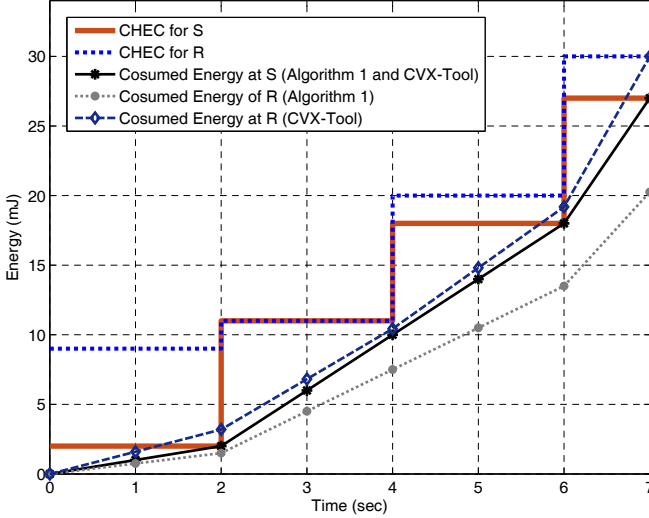


Fig. 4. Comparison between the allocated powers for S and R in our greedy algorithm and that of the CVX-Tool in Example 1.

Now, we present an example that applies to the considered case.

Example 1: Suppose that in time instants 0, 2, 4 and 6 sec, the S and R harvest energies $\mathbf{E}_1 = [2, 9, 7, 9]$ mJ and $\mathbf{E}_2 = [9, 2, 9, 10]$ mJ, respectively. We assume $a = b = 2$, $T = 7$. Also, we use *CVX-Tool*, a package for solving disciplined convex programs [36] to solve this problem numerically. The CVX-Tool allocates the power for S and R as $\mathbf{P}_1^* = [1, 4, 9]$ mW with duration $\mathbf{L}_1^* = [2, 4, 1]$ and $\mathbf{P}_2^* = [1.5970, 3.6188, 4.3775, 4.3795, 10.8113]$ mW with duration $\mathbf{L}_2^* = [2, 2, 1, 1, 1]$. Algorithm 1, on the other hand, assigns $\mathbf{P}_1^* = [1, 4, 9]$ mW with duration $\mathbf{L}_1^* = [2, 4, 1]$ and $\mathbf{P}_2^* = [0.75, 3, 6.75]$ mW with duration $\mathbf{L}_2^* = [2, 4, 1]$ for S and R, respectively. The above two power allocations provide the same total transmitted bits with the difference that Algorithm 1 uses the least possible power for the relay. This causes to leave some energies unused ($E_{\text{excess}} = 9.75$ mJ in this example). This excess energy can be stored in the R for future use or sending its own data in cooperative scenarios. The harvested energies and the consumed energies of the S and R in our proposed greedy algorithm and those provided in CVX-Tool are shown in Fig. 4.

Example 1 shows the following Lemma.

Lemma 3: Under the optimal policy of NC-EH-RC, if powers of S or R changes in an instant, the total harvested energy in the previous epochs of that node has not necessarily been consumed completely by this instant. Thus, [16, Lemma 3] does not hold for optimal policy of NC-EH-RC.

Corollary 1: The power allocation for S and R based on the *disjoint optimization*, which follows the separate shortest-path algorithm for S and R, constructs a sub-optimal solution for NC-EH-RC.

Proof: This follows directly using Lemma 3. In other word, [16, Lemma 3] which is a necessary condition for optimality of disjoint optimization algorithm (shortest path) in S and R, does not hold in general. ■

Remark 3: In section VI, we give an example where in

the optimal allocation policy, the power of nodes are not monotonically increasing. Therefore, [16, Lemma 1] does not hold for the general optimal solution of NC-EH-RC.

Remark 4: The disjoint optimization for the S and R is optimal for the special cases studied in [28]. Those cases that are presented for coherent DF Gaussian RC can be used for NC-EH-RC, as well.

In the following, we propose a suboptimal algorithmic solutions for the power allocation problem in NC-EH-RC. This solution is optimal for some EH realizations of S and R. We present some numerical examples in section VI to study the scenarios in which this suboptimal solution is optimal.

B. Suboptimal Algorithmic Solution for NC-EH-RC with No ET

Here, we come up with a novel approach to find a suboptimal power allocation by introducing a new constraint on the total transmission powers of the nodes. First, we consider a simplified problem, which its solution provides a suboptimal solution to our problem. Then, we propose an algorithm which solves the simplified problem.

This suboptimal power allocation for NC-EH-RC is provided in Algorithm 2. Unlike Algorithm 1, this algorithm solves the problem by assuming that the cost function is dominated by the first term under the minimum of (4). If this assumption does not hold, the solution will be suboptimal. We add the following constraint to the problem by combining (10) and (11):

$$\sum_{i=1}^k \tilde{P}_t^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_t^i, \quad \forall k, \quad (34)$$

where $\tilde{P}_t^i = P_1^i + b^2 P_2^i$, $\forall i$ and $\tilde{E}_t^i = E_1^i + b^2 E_2^i$, $\forall i$.

Therefore, the problem is as follows

$$\begin{aligned} \max_{P_1, P_2} \quad & \sum_{i=1}^K C \left(\frac{P_1^i + b^2 P_2^i}{N_0} \right) l^i \\ \text{s.t.} \quad & (9) - (11), (34) \end{aligned} \quad (35)$$

By relaxing constraints (10) and (11) from the above problem, we get

$$\begin{aligned} \max_{\tilde{P}_t} \quad & \sum_{i=1}^K C \left(\frac{\tilde{P}_t^i}{N_0} \right) l^i \\ \text{s.t.} \quad & \tilde{P}_t^i \geq 0, (34) \end{aligned}$$

It can be easily seen that the solution for $\tilde{\mathbf{P}}_t$ in the above problem follows the shortest-path algorithm. Now, to find the solution to the problem (35), it suffices to allocate P_1^{i*}, P_2^{i*} , $\forall i$ satisfying (9)-(11) and $\tilde{P}_t^{i*} = P_1^{i*} + b^2 P_2^{i*}$, $\forall i$. One solution for this problem is presented in Algorithm 2. Note that if at any time instant, (34) is satisfied with equality, then (10) and (11) should also be satisfied with equality. In other words, if the total harvested energies of the network is completely used up in an instant, the same should be happened for energies of S and R. Hence, we force the S and R to empty their batteries whenever the (34) is active. Within such instants,

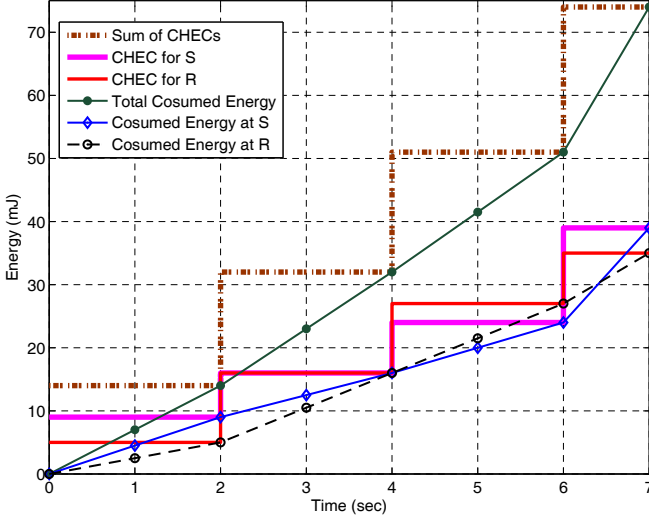


Fig. 5. An example with modified shortest path power allocation for S and R. Total consumed energy follows the shortest path.

we allocate power for S and R individually based on shortest path algorithm. This allocation for S and R is called *modified shortest path*. It is obvious that the power allocations for S and R are surely feasible. Fig. 5 shows a typical example, in which \mathbf{P}_1 and \mathbf{P}_2 are obtained using Algorithm 2. In section VI, we present some examples that this suboptimal solution is optimal.

Algorithm 2 Suboptimal power allocation for NC-EH- \mathcal{RC} with no ET

Total Power Allocation

(1) Merge the harvested energies of S and R to produce total harvested energy as $\tilde{E}_t^i = E_1^i + b^2 E_2^i$, $i = 1, \dots, K+1$.

(2) Find optimal total power allocation as

$$o_v = \underset{o_v-1 < i \leq K+1}{\operatorname{argmin}} \frac{\sum_{j=o_v-1}^{i-1} \tilde{E}_t^j}{t^i - t^{o_v-1}}$$

$$\tilde{P}_t^{o_v*} = \frac{\sum_{j=o_v-1}^{o_v-1} \tilde{E}_t^j}{t^{o_v} - t^{o_v-1}}$$

(3) Partition transmission time into time slots that total power is fixed (or (34) is active), i.e., s_j , $j = 1, \dots, Q$ ($\sum_{j=1}^Q s_j = T$).

for $j = 1$ **to** Q **do**

Individual Power Allocation for S and R

(4) Allocate power for S in time slot s_j according to single-user shortest path algorithm.

(5) Allocate power for R in time slot s_j according to single-user shortest path algorithm.

end for

IV. NC-EH- \mathcal{RC} WITH ONE-WAY ENERGY TRANSFER FROM S TO R

In this section, we concentrate on NC-EH- \mathcal{RC} with one-way ET from S to R, as in [30], and studying the optimal algorithmic solutions for power allocation problem. The system model in this case is depicted in Fig. 6. We first formulate the

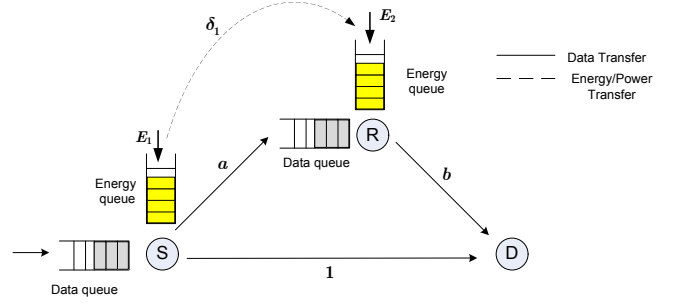


Fig. 6. Full-duplex RC with energy harvesting S and R and one-way ET from S to R.

problem as follows:

$$\max_{\{P_1^i\}\{\tilde{P}_2^i\}\{\delta_1^i\}} \sum_{i=1}^{K+1} \left(\lambda^i \mathcal{C} \left(\frac{P_1^i + \tilde{P}_2^i}{N_0} \right) + (1 - \lambda^i) \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right) l^i \quad (36)$$

$$s.t. \quad P_1^i \geq 0, \tilde{P}_2^i \geq 0, \quad i = 1, \dots, K+1, \quad (37)$$

$$\delta_1^i \geq 0 \quad i = 1, \dots, K+1, \quad (38)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} E_1^i - \delta_1^i, \quad k = 1, \dots, K+1, \quad (39)$$

$$\sum_{i=1}^k \tilde{P}_2^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_2^i + \delta_1^i, \quad k = 1, \dots, K+1, \quad (40)$$

where δ_1^i denotes the energy transfer at epoch i from S to R, $\tilde{P}_2 = b^2 P_2$ and $\tilde{E}_2 = b^2 E_2$. Then, we modify the cost function as

$$C \geq \min \left\{ \mathcal{C} \left(\frac{\tilde{P}_t}{N_0} \right), \mathcal{C} \left(\frac{[a^2]^\dagger P_1}{N_0} \right) \right\} \quad (41)$$

where $\tilde{P}_t = \mathbf{P}_1 + \tilde{\mathbf{P}}_2$.

Lemma 4: We can replace energy causality constraints for R in (40), with the following

$$\sum_{i=1}^k \tilde{P}_t^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_t^i, \quad \forall k \quad (42)$$

Proof: See Appendix A. ■

Remark 5: We consider $\tilde{P}_t = \mathbf{P}_1 + b^2 \mathbf{P}_2$ instead of $\mathbf{P}_t = \mathbf{P}_1 + \mathbf{P}_2$ to define total power. Note that a feasible P_1^i and P_2^i for $\sum_{i=1}^k \tilde{P}_t^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_t^i, \forall k$ does not necessarily satisfy $\sum_{i=1}^k P_t^i l^i \leq \sum_{i=0}^{k-1} E_t^i, \forall k$. In other words, any feasible partitioning of \tilde{P}_t^i into P_1^i and P_2^i is not necessarily a feasible partitioning for P_t^i .

Applying Lemma 4, (36)-(40) are expressed as

$$\max_{\{P_1^i\}\{\tilde{P}_2^i\}\{\delta_1^i\}} \sum_{i=1}^{K+1} \left(\lambda^i \mathcal{C} \left(\frac{\tilde{P}_t^i}{N_0} \right) + (1 - \lambda^i) \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right) l^i \quad (43)$$

$$s.t. \quad P_1^i \geq 0, \tilde{P}_t^i - P_1^i \geq 0, \delta_1^i \geq 0 \quad i = 1, \dots, K+1, \quad (44)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} E_1^i - \delta_1^i, \quad k = 1, \dots, K+1, \quad (45)$$

$$\sum_{i=1}^k \tilde{P}_t^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_t^i, \quad k = 1, \dots, K+1, \quad (46)$$

It can be easily observed that this problem is convex. Thus, the Lagrangian is computed as

$$\begin{aligned} L = & \sum_{i=1}^{K+1} \left(\lambda^i \mathcal{C} \left(\frac{\tilde{P}_t^i}{N_0} \right) + (1 - \lambda^i) \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right) l^i \\ & - \sum_{k=1}^K \xi_k \left(\sum_{i=1}^k P_1^i l^i - \sum_{i=0}^{k-1} (E_1^i - \delta_1^i) \right) \\ & - \sum_{k=1}^K \mu_k \left(\sum_{i=1}^k \tilde{P}_t^i l^i - \sum_{i=0}^{k-1} \tilde{E}_t^i \right) \\ & + \sum_{i=1}^{K+1} \vartheta_i P_1^i + \sum_{i=1}^{K+1} \eta_i (\tilde{P}_t^i - P_1^i) + \sum_{i=1}^{K+1} \phi_i \delta_1^i. \end{aligned} \quad (47)$$

KKT optimality conditions for (47) are

$$\frac{(1 - \lambda^i)[a^2]^\dagger}{[a^2]^\dagger P_1^i + N_0} - \sum_{k=i}^K \xi_k + \vartheta^i - \eta^i = 0, \quad \forall i \quad (48)$$

$$\frac{\lambda^i}{\tilde{P}_t^i + N_0} - \sum_{k=i}^K \mu_k + \eta_i = 0, \quad \forall i \quad (49)$$

$$- \sum_{k=i}^K \xi_k + \phi^i = 0, \quad \forall i \quad (50)$$

with the following complementary slackness conditions

$$\mu_k \left(\sum_{i=1}^k \tilde{P}_t^i l^i - \sum_{i=0}^{k-1} \tilde{E}_t^i \right) = 0, \quad \forall k \quad (51)$$

$$\xi_k \left(\sum_{i=1}^k P_1^i l^i - \sum_{i=0}^{k-1} (E_1^i - \delta_1^i) \right) = 0, \quad \forall k \quad (52)$$

$$\vartheta_i P_1^i = 0, \quad \forall k \quad (53)$$

$$\eta_i (\tilde{P}_t^i - P_1^i) = 0, \quad \forall k \quad (54)$$

$$\phi_i \delta_1^i = 0, \quad \forall k \quad (55)$$

The results for optimal allocated powers are as follows

$$\tilde{P}_t^{i*} = \frac{\lambda^i}{\sum_{k=i}^K \mu_k} - N_0, \quad \forall i \quad (56)$$

and

$$P_1^{i*} = \frac{(1 - \lambda^i)}{\sum_{k=i}^K \xi_k} - \frac{N_0}{[a^2]^\dagger}, \quad \forall i \quad (57)$$

and finally

$$P_2^{i*} = \frac{\tilde{P}_t^{i*} - P_1^{i*}}{b^2}, \quad \forall i. \quad (58)$$

Considering (44), the allocated powers for nodes must be nonnegative. However, there is no incentive to let $P_1^i = 0$ or

$P_2^i = 0$ for any i . This is due to the fact that $E_1^1 > 0$ and $E_2^1 > 0$ (see [17] for more details). Thus, the complementary slackness conditions ($\vartheta_i P_1^i = 0, \forall i$ and $\eta_i (\tilde{P}_t^i - P_1^i) = 0, \forall i$) dictates $\vartheta_i = \eta_i = 0, \forall i$.

Now, we assume that S is in good EH condition.

Definition 2: We define the class of problem (36)-(40) with S in good EH condition as the problems, where $P_1^{i*}, P_2^{i*}, \forall i$ (in optimal solution) satisfy

$$\tilde{C}_2^i(P_1^{i*}) \geq \tilde{C}_1^i(P_1^{i*}, P_2^{i*}), \quad \forall i \quad (59)$$

$$\forall k, \quad \exists \delta_1^{i*} \geq 0 \text{ s.t. } \sum_{i=1}^k P_1^{i*} l^i \leq \sum_{i=0}^{k-1} E_1^i - \delta_1^{i*} \quad (60)$$

This means that S not only scavenged sufficient energy to use for its transmission, but also it can provide energy for R by transferring some parts of its harvested energy.

Here, MAC bound (\tilde{C}_1) is the bottleneck. The optimal allocation, for the case when S is in good EH condition, is given in Algorithm 3.

Lemma 5: Algorithm 3 present the optimal algorithmic solution for power allocation problem of NC-EH-RC with one-way ET from S to R, when S is in good EH condition.

Proof: When S is in good EH condition, according to Definition 2, the cost function is $\tilde{C}_1(\tilde{\mathbf{P}}_t)$. Therefore, problem (43) reduces to

$$\max_{\{\tilde{P}_t^i\}} \sum_{i=1}^{K+1} \mathcal{C} \left(\frac{\tilde{P}_t^i}{N_0} \right) l^i \quad (61)$$

$$s.t. \quad \tilde{P}_t^i \geq 0, \quad i = 1, \dots, K+1, \quad (62)$$

$$\sum_{i=1}^k \tilde{P}_t^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_t^i, \quad k = 1, \dots, K+1, \quad (63)$$

This is equivalent to inserting $\lambda^i = 1, \forall i$ in (43). In this case, substituting $\lambda^i = 1, \forall i$ in (56), we have

$$\tilde{P}_t^{i*} = \frac{1}{\sum_{k=i}^K \mu_k} - N_0, \quad \forall i. \quad (64)$$

It is clear that the optimal solution for $\tilde{\mathbf{P}}_t$ follows the shortest-path algorithm (we call it $\tilde{\mathbf{P}}_t^*$). This is due to the fact that three necessary lemmas for this conclusion [16, Lemmas 1, 2 and 3] can be deduced using (64). The optimal point is

$\mathcal{B}^* = \sum_{i=1}^{K+1} \mathcal{C}(\tilde{P}_t^{i*}/N_0) l^i$ for $\tilde{\mathbf{P}}_t^* = \mathbf{P}_1^* + b^2 \mathbf{P}_2^*$ and feasible \mathbf{P}_1^* and \mathbf{P}_2^* , irrespective of exact values for P_1^{i*} and P_2^{i*} . We set $\tilde{P}_t^{i*} = [a^2]^\dagger P_1^{i*}, \forall i$, which satisfies (59). Substitution in (60) yields: $\forall k, \quad \exists \delta_1^{i*} \geq 0, \forall i$ such that $\sum_{i=1}^k \frac{\tilde{P}_t^{i*}}{[a^2]^\dagger} l^i \leq \sum_{i=0}^{k-1} E_1^i - \delta_1^{i*}, \forall k$. Therefore, our optimal allocations are as

$$P_1^{i*} = \frac{\tilde{P}_t^{i*}}{[a^2]^\dagger}, \quad P_2^{i*} = \frac{([a^2]^\dagger - 1) \tilde{P}_t^{i*}}{[a^2]^\dagger b^2}, \quad \forall i$$

This allocation is feasible due to Definition 2. Note that this partitioning makes \tilde{C}_1 and \tilde{C}_2 equal. We remark that when S

is in good EH condition, Algorithm 3 never enters the **return** line. This means that the **if** condition is always met. This completes the proof. ■

Algorithm 3 Optimal power allocation algorithm for NC-EH- \mathcal{RC} with one-way ET from S to R

Total Power Allocation

(1) Set $\tilde{E}_t^i = E_1^i + b^2 E_2^i$, $i = 1, \dots, K + 1$.

(2) Find optimal power allocation for $\tilde{\mathbf{P}}_t$ as

$$o_v = \underset{o_{v-1} < i \leq K+1}{\operatorname{argmin}} \frac{\sum_{j=o_{v-1}}^{i-1} \tilde{E}_t^j}{t^i - t^{o_{v-1}-1}}$$

$$\tilde{P}_t^{v*} = \frac{\sum_{j=o_{v-1}}^{o_v-1} \tilde{E}_t^j}{t^{o_v} - t^{o_{v-1}-1}}$$

Feasibility Problem for Power Allocation at S

if $\forall k, \exists \delta_1^{i*} \geq 0$ s.t. $\sum_{i=1}^k \frac{\tilde{P}_t^{i*}}{[a^2]^\dagger} l^i \leq \sum_{i=0}^{k-1} E_1^i - \delta_1^{i*}$ **then**

Individual Power Allocation for S and R

(3) Find optimal power allocation for S as $P_1^{v*} = \frac{\tilde{P}_t^{v*}}{[a^2]^\dagger}$

(4) Find optimal power allocation for R as $P_2^{v*} = \frac{([a^2]^\dagger - 1) \tilde{P}_t^{v*}}{[a^2]^\dagger b^2}$

else

return The algorithmic optimal solution is not known in general.

end if

Remark 6: Consider a situation where the harvested energy at S (i.e., \mathbf{E}_1) is sufficiently large, so that optimal solution of (43) is obtained while (45) is inactive for all epochs. Using slackness condition in (52), we reach $\xi_k = 0$, $\forall k$. Substituting this in KKT optimality condition in (50), we obtain $\phi_i = 0$, $\forall i$. Combining with (48), we conclude that $\lambda^i = 1$, $\forall i$. Therefore, in this case, the problem (43)-(46) is transformed to (61)-(63). Thus, sufficiently large \mathbf{E}_1 is a special case of S in good EH condition as expected and hence Algorithm 3 is optimal in this case.

Lemma 6: In the optimal solution provided by Algorithm 3, we have the followings

$$\sum_{i=1}^{K+1} P_1^{i*} l^i = \sum_{i=0}^K E_1^i - \delta_1^{i*} \quad (65)$$

$$\sum_{i=1}^{K+1} P_2^{i*} l^i = \sum_{i=0}^K E_2^i + \frac{\delta_1^{i*}}{b^2} \quad (66)$$

This means that S must completely use up its total harvested energy either for transferring toward R or utilizing it for data transmission. On the other hand, R has to use up total energies received by S and harvested through environment by the end of transmission time.

Proof: As proved in Lemma 5, when S is in good EH condition, the cost function is only expressed in \tilde{C}_1 , which is a monotonically increasing function of \mathbf{P}_2 . If the constraints in (65) and (66) are satisfied with strict inequalities in the optimal solution, we can increase δ_1^{K*} without violating (45). So, we can increase P_2^{K+1*} , as well. With this increment, \tilde{C}_1 increases. This contradicts the optimality of P_1^{i*} , P_2^{i*} and δ_1^{i*} . ■

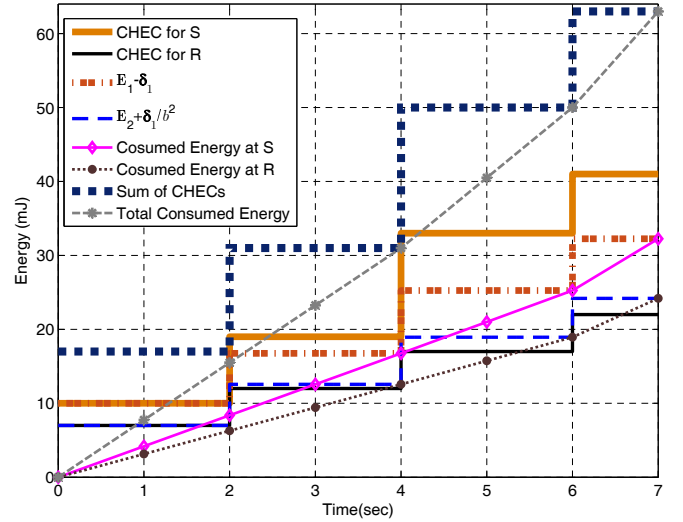


Fig. 7. Optimal algorithmic solution for the case that one-way ET from S to R is possible.

Here, an example is presented that optimal solution is obtained using algorithm 3.

Example 2: We assume that S and R harvest $\mathbf{E}_1 = [10, 9, 14, 8]$ mJ and $\mathbf{E}_2 = [7, 5, 5, 5]$ mJ, respectively at time instants $t = [0, 2, 4, 6]$ sec. Time duration of interest is $T = 7$ sec and we set $a = 2$ and $b = 2$. This example is the case that there is a positive one-way ET vector $\delta_1^* = [0, 2.25, 5.5, 1]$ mJ at time instants $t = [0, 2, 4, 6]$, for which the optimal allocation using algorithm 3 is possible. This algorithm assigns the power of S and R as $\mathbf{P}_1 = [4.1875, 4.25, 7]$ mW and $\mathbf{P}_2 = [3.1406, 3.1875, 5.25]$ mW, respectively, with durations $\mathbf{L}_1 = \mathbf{L}_2 = [4, 2, 1]$ sec. Figure 7 shows the energy arrivals at S and R and their allocated powers using Algorithm 3. Besides, total harvested energy curves and optimum total power, which is allocated based on shortest path algorithm, are shown in this figure. The allocated power of S is restricted to $\mathbf{E}_1 - \delta_1^*$ and that of R is restricted to $\mathbf{E}_2 + \delta_1^{i*}/b^2$, as expected.

V. NC-EH- \mathcal{RC} WITH TWO-WAY ENERGY TRANSFER

The NC-EH- \mathcal{RC} with two-way ET, in which S and R share their harvested energies with each other is shown in Fig. 8. This is a case, where we find the general algorithmic optimal solution. In this case the problem is as follows

$$\max_{P_1, P_2, \delta_1, \delta_2} \sum_{i=1}^{K+1} \min \left\{ \mathcal{C} \left(\frac{P_1^i + \tilde{P}_2^i}{N_0} \right), \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right\} l^i \quad (67)$$

$$\text{s.t. } P_1^i \geq 0, \tilde{P}_2^i \geq 0, \quad i = 1, \dots, K + 1, \quad (68)$$

$$\delta_1^i \geq 0, \delta_2^i \geq 0, \quad i = 1, \dots, K + 1, \quad (69)$$

$$\delta_1^i \cdot \delta_2^i = 0, \quad i = 1, \dots, K + 1, \quad (70)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} E_1^i - \delta_1^i + \delta_2^i, \quad k = 1, \dots, K + 1, \quad (71)$$

$$\sum_{i=1}^k \tilde{P}_2^i l^i \leq \sum_{i=0}^{k-1} \tilde{E}_2^i - \delta_2^i + \delta_1^i, \quad k = 1, \dots, K + 1, \quad (72)$$

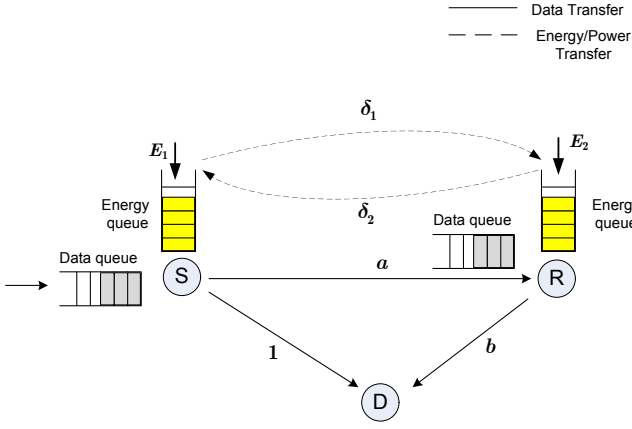


Fig. 8. Full-duplex RC with two-way ET. S and R share their harvested energies in order to have better control on the network resources.

where δ_1^i and δ_2^i denote the energy transfers in epoch i in $S \rightarrow R$ and $R \rightarrow S$ directions, respectively. The constraint (70) arises due to the fact that it does not make sense to send and receive energy at the same time. We call this constraint *half-duplex energy transfer constraint* in each epoch. This problem is not convex due to (70). Now, we transform it into a convex optimization problem.

Lemma 7: The problem in (67)-(72) is equivalent to the following convex optimization problem:

$$\max_{P_1, P_2} \sum_{i=1}^{K+1} \min \left\{ c \left(\frac{\tilde{P}_t^i}{N_0} \right), c \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) \right\} t^i \quad (73)$$

$$\text{s.t.} \quad P_1^i \geq 0, \tilde{P}_t^i \geq 0, \quad i = 1, \dots, K+1, \quad (74)$$

$$\sum_{i=1}^k \tilde{P}_t^i t^i \leq \sum_{i=0}^{k-1} \tilde{E}_t^i, \quad k = 1, \dots, K+1, \quad (75)$$

This means that two problems have the same optimal values.

Proof: See Appendix B. ■

Theorem 1: Algorithm 4 provides the optimal algorithmic solution for the power allocation problem in NC-EH-RC with two-way ET between S and R, presented in (67)-(72).

Outline of Proof: We transform the problem in (67)-(72) to the one in (73)-(75), using Lemma 7. This shows that only feasibility on total power must be met and any P_1^{i*} and P_2^{i*} satisfying (77) are feasible. We set $\tilde{P}_t^i = [a^2]^\dagger P_1^i, \forall i$. This leads to the following allocations:

$$P_1^v = \frac{\tilde{P}_t^v}{[a^2]^\dagger}, \quad P_2^v = \frac{([a^2]^\dagger - 1)\tilde{P}_t^v}{[a^2]^\dagger b^2}, \quad \forall v \quad (76)$$

It can be easily seen that in this case the problem is only expressed in terms of \tilde{P}_t . Therefore, we first optimally allocate network's total power based on optimal allocation for point-to-point channel (shortest path algorithm). Then, we partition the total power as in (76). As indicated in the proof of Lemma 7, optimal allocation is not unique congruent to the previous parts. ■

Example 3: In order to show the performance of Algorithm 4, we assume that S and R harvest energy at time instants $t = [0, 2, 4, 6]$, with the amounts of $\mathbf{E}_1 = [10, 9, 7, 9]$ mJ

Algorithm 4 Optimal power allocation for NC-EH-RC with two-way ET

Total Power Allocation

(1) Set $\tilde{E}_t^i = E_1^i + b^2 E_2^i, i = 1, \dots, K+1$.

(2) Find optimal power allocation for \tilde{P}_t as

$$o_v = \underset{o_{v-1} < i \leq K+1}{\operatorname{argmin}} \frac{\sum_{j=o_{v-1}}^{i-1} \tilde{E}_t^j}{t^i - t^{o_{v-1}}}$$

$$\tilde{P}_t^{v*} = \frac{\sum_{j=o_v-1}^{o_v-1} \tilde{E}_t^j}{t^{o_v} - t^{o_v-1}}$$

Individual Power Allocation for S and R

(3) Find optimal power allocation for S as $P_1^{v*} = \frac{\tilde{P}_t^{v*}}{[a^2]^\dagger}$

(4) Find optimal power allocation for R as $P_2^{v*} = \frac{([a^2]^\dagger - 1)\tilde{P}_t^{v*}}{[a^2]^\dagger b^2} =$

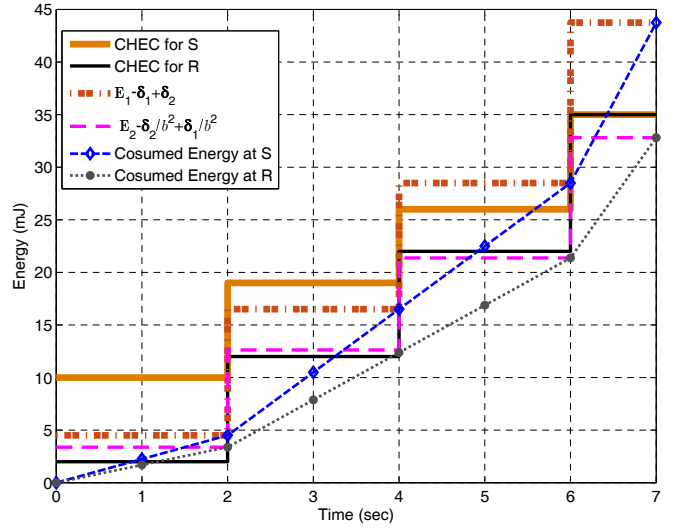


Fig. 9. Optimal algorithmic solution for NC-EH-RC with two-way ET in Example 3.

and $\mathbf{E}_2 = [2, 10, 10, 13]$ mJ, respectively. Other parameters are $T = 7$ sec, $a = 2$ and $b = 2$. Algorithm 4 allocates $\mathbf{P}_1 = [2.25, 6, 15.25]$ mW and $\mathbf{P}_2 = [1.6875, 4.5, 11.4375]$ mW with durations $\mathbf{L}_1 = \mathbf{L}_2 = [2, 4, 1]$ sec for S and R, respectively. This needs the energy transfer of $\delta_1^* = [5.5, 0, 0, 0]$ mJ from S to R and energy transfer of $\delta_2^* = [0, 3, 5, 6.25]$ mJ from R to S at time instants $t = [0, 2, 4, 6]$. These values are shown in Fig. 9. The curves associated with $\mathbf{E}_1 - \delta_1^* + \delta_2^*$ and $\mathbf{E}_2 - \delta_2^*/b^2 + \delta_1^*/b^2$ are also shown, which are the upper limits of energy consumption in nodes.

Lemma 8: General optimal algorithmic solution for nodes in NC-EH-RC with two-way ET presented in Algorithm 4 is equivalent to disjoint optimization for S and R with modified EH pattern \mathfrak{E}_1 and \mathfrak{E}_2 , where $\mathfrak{E}_1 = \mathbf{E}_1 - \delta_1^* + \delta_2^*$ and $\mathfrak{E}_2 = \mathbf{E}_2 - \delta_2^*/b^2 + \delta_1^*/b^2$.

Proof: See Appendix C. ■

VI. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we investigate the performance of our proposed power allocation algorithms for NC-EH-RC. They consist of optimal and suboptimal solutions that are optimal for some special cases and are presented for NC-EH-RC with

no ET, one-way ET from S to R and two-way ET. We consider a band-limited AWGN channel with noise power spectral density $N_0 = 10^{-19}$ W/Hz and bandwidth $W = 1$ MHz. The distances among nodes are assumed to be 1 Km and path loss is $\psi = 100$ dB (typical values used in some EH literatures, e.g. [16]). Therefore, the Channel to Noise Ratio (CNR) of links in NC-EH- \mathcal{RC} are $\gamma_{sr} = \frac{a^2\psi}{N_0W}$, $\gamma_{rd} = \frac{b^2\psi}{N_0W}$ and $\gamma_{sd} = \frac{\psi}{N_0W}$ for S-R, R-D and S-D links, respectively. The channel gains are set to $a = 2$ and $b = 2$ for S-R and R-D links, respectively. Harvesting time instants are $t = [0, 2, 4, 6]$ sec with $T = 7$ sec as the time duration of interest. Harvested energies at S and R are samples of Poisson distribution with mean $\bar{E}_1 = \bar{E}_2 = 10$ mJ.

In Table I, the performance of our two proposed suboptimal allocation algorithms (Algorithm 2 and disjoint optimization algorithm), and optimal allocation (Algorithm 4) is evaluated and compared with optimal numerical solutions presented by CVX-Tool. Six scenarios are studied: (s1) Scenario 1 is the case that Algorithm 2 outperforms the disjoint optimization algorithm, while both of them are suboptimal. (s2) Scenario 2 is the same as (s1) with the difference that Algorithm 2 is optimal. (s3) Scenario 3, is the case where disjoint optimization outperform Algorithm 2, while both solutions are suboptimal. (s4) Scenario 4 presents an example that disjoint optimization is optimal and has better performance compared to that of Algorithm 2. (s5) In scenario 5, two algorithms have the same performance, which are suboptimal. (s6) Scenario 6 is the case that both algorithmic solutions are optimal. Note that (s2) and (s3) correspond to the examples 2 and 3 in sections IV and V, respectively. For these scenarios, optimal allocated powers in Algorithms 3 and 4 are shown in Fig. 7 and Fig. 9, respectively.

In Table I, the results of optimal CVX-Tool are provided for the cases that one-way ET from S to R and two-way ET between S and R are possible. In (s1), transferring energy from S to R improves the performance but not vice versa. This follows from the fact that the EH at S is better than the EH at R. Thus, energy is required in R more than S. In (s2), we gain nothing by ET (EH at S and R is well equalized and sharing does not improves the performance), whereas in (s3), ET provides the opportunity for better utilization of network energy resources. Here, we achieve better performance, with more capable nodes sharing energy bi-directionally, compared to the no ET and one-way ET cases. In scenarios 4, 5 and 6, unlike (s1), the performance improves only when R is able to transfer some part of its energy toward S. We note that with ET capability added to transmitting nodes, we are able to provide algorithmic optimal solution for the cases that are hard to achieve without them. We remind that since Algorithm 4 presents the optimal solution for the case with two-way ET, it leads to the same result as CVX-Tool (see the last column of Table I), but with different power allocations which are given in Tables II, III, and IV.

Tables II, III, and IV provide the powers allocated to S and R in different solution methods for scenarios 2, 4, and 6, respectively. In Table II, where Algorithm 2 is optimal, R's allocated powers are [3, 2.5, 5] mJ with durations [4, 2, 1] sec.

On the other hand, in disjoint optimization (suboptimum in this example), they are [2.8333, 5] mJ with durations [6, 1] sec. Therefore, optimal powers do not have necessarily monotonic behaviour. Note that the CVX-Tool shows the same behaviour as Algorithm 2. Unlike (s2), Table III shows the case where equalized power of S in disjoint optimal algorithm, i.e., $P_1 = 5.4286$ mW with duration $L_1 = 7$ sec, provides optimal performance. This exceeds the performance of Algorithm 2 ($P_1 = [5.5, 5]$ mW with durations $L_1 = [6, 1]$ sec for S with same power allocation for R). In Table IV, where two suboptimal allocation algorithms are optimal, power allocations are exactly the same. Also, we observe in these tables that even though Algorithm 4 provides the same optimal value as the CVX-Tool with two-way ET, its allocated powers are totally different. We see in three scenarios of Tables II, III, and IV that the power allocation of Algorithm 4 is more fair compared to diverse power allocation of CVX-Tool. Besides, high power allocation for S in CVX-Tool may cause some technical difficulties, if utilized in practice. This highlights the applicability of Algorithm 4 in practical transmitter schedulers.

VII. CONCLUSION

We investigated the optimal power allocation for a three-node full-duplex non-coherent decode-and-forward Gaussian relay channel with energy harvesting source and relay nodes (called NC-EH- \mathcal{RC}). Three cases were considered based on the capability of the source and the relay nodes to transfer parts of their energies to each other, namely no ET, one-way ET and two-way ET. The original problem for NC-EH- \mathcal{RC} with no ET has a complicated min-max form, which is not easy to solve. We showed that it is transformed to a tractable convex optimization problem, which can be solved efficiently. However, convex optimization did not provide any structural property of optimal solution to be used in devising algorithmic solutions. Following a different perspective, we studied cases where optimal algorithmic solutions are found. These cases were investigated to give insight by revealing some key specifications of general optimal solution. These specifications discriminated our problem from the others in the existing works and showed that our problem can not be reduced to the existing EH problems. Also, we proposed some suboptimal algorithmic solutions that are optimal for some realizations of EH pattern at S and R. Moreover, in NC-EH- \mathcal{RC} with one-way ET, we found a class of problems, where the optimal algorithmic solution was devised. For NC-EH- \mathcal{RC} with two-way ET, we derived some interesting properties of optimal solution that are used to find optimal algorithmic solution *in general*. Besides, the performance of our proposed algorithms were evaluated numerically and compared with optimal numerical convex optimization tools. Numerical results highlighted the applicability of our proposed algorithms in practical transmitter schedulers.

APPENDIX A PROOF OF LEMMA 4

It is obvious that for a feasible solution satisfying (39) and (40), (42) is also held by combining (39) and (40). To show

TABLE I

TOTAL TRANSMITTED BITS (IN MBITS) IN THE SUBOPTIMAL ALGORITHMS, WHICH ARE PROPOSED FOR NC-EH- \mathcal{RC} . OPTIMAL POWER ALLOCATION USING CONVEX OPTIMIZATION TOOL IS INCLUDED FOR NO-ET, ONE-WAY ET AND TWO-WAY ET. THE RESULTS OF OPTIMAL POWER ALLOCATION IN ALGORITHM 4 IS ALSO INCLUDED.

Scenarios	EH values for S, \mathbf{E}_1				EH values for R, \mathbf{E}_2				Algorithm 2	Disjoint Optimization Algorithm	CVX-Tool [36]	CVX-Tool with one-way ET	CVX-Tool with two-way ET, and Algorithm 4
Scenario 1	[10	21	14	9]	[7	5	8	11]	31.8339	31.8082	32.1965	32.4212	32.4212
Scenario 2	[10	9	14	8]	[7	5	5	5]	29.7968	29.7821	29.7968	29.7968	29.7968
Scenario 3	[10	9	7	9]	[2	10	10	13]	28.2032	28.4398	28.9548	29.8207	31.1735
Scenario 4	[17	7	9	5]	[13	7	9	10]	31.5337	31.5387	31.5387	31.5387	33.6705
Scenario 5	[7	11	15	15]	[12	15	10	8]	32.3543	32.3543	32.7000	32.7000	35.3402
Scenario 6	[7	11	11	9]	[10	7	11	12]	31.1175	31.1175	31.1175	31.1175	33.4912

TABLE II

POWER ALLOCATION FOR S AND R IN NC-EH- \mathcal{RC} . THE RESULTS ARE PRESENTED FOR SCENARIO 2 OF TABLE I, IN WHICH ALGORITHM 2 IS OPTIMAL. POWER ALLOCATION IN ALGORITHM 4 IS ALSO INCLUDED TO BE COMPARED WITH THAT OF CVX-TOOL WITH TWO-WAY ET.

Scenario	Solution Methods	Power Allocation for S, \mathbf{P}_1 and \mathbf{L}_1					Power Allocation for R, \mathbf{P}_2 and \mathbf{L}_2						
Scenario 2	Algorithm 2	\mathbf{P}_1	=	[4.75	7	8]	\mathbf{P}_2	=	[3	2.5	5]
		\mathbf{L}_1	=	[4	2	1]	\mathbf{L}_2	=	[4	2	1]
	Disjoint Optimization Algorithm	\mathbf{P}_1	=	[4.75	7	8]	\mathbf{P}_2	=	[2.8333	5]
		\mathbf{L}_1	=	[4	2	1]	\mathbf{L}_2	=	[6	1]
	CVX-Tool	\mathbf{P}_1	=	[4.6569	4.8430	7	8]	\mathbf{P}_2	=	[3.0233	2.9767	2.5	5]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]
	CVX-Tool with one-way ET	\mathbf{P}_1	=	[4.6709	4.7416	6.2051	9.7647]	\mathbf{P}_2	=	[3.0198	3.0021	2.6987	4.5588]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]
	Algorithm 4	\mathbf{P}_1	=	[4.1875	4.25	7]	\mathbf{P}_2	=	[3.1406	3.1875	5.25]
		\mathbf{L}_1	=	[4	2	1]	\mathbf{L}_2	=	[4	2	1]
	CVX-Tool with two-way ET	\mathbf{P}_1	=	[14.336	14.2945	14.4894	23.7573]	\mathbf{P}_2	=	[0.6035	0.6139	0.6276	1.0607]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]

the converse, if (42) is satisfied for an arbitrary $i = \check{i}$, then

$$\sum_{i=1}^{\check{i}} (P_1^i + \tilde{P}_2^i) l^i \leq \sum_{i=1}^{\check{i}} E_1^{i-1} + \tilde{E}_2^{i-1},$$

Subtracting $P_1^i l^i > 0, \forall i$ from both sides of the above inequality, we have

$$\sum_{i=1}^{\check{i}} \tilde{P}_2^i l^i \leq \sum_{i=1}^{\check{i}} \tilde{E}_2^{i-1} + E_1^{i-1} - P_1^i l^i,$$

Since $E_1^{i-1} - P_1^i l^i \geq 0$ according to (38)-(39), defining $\delta^i = E_1^i - P_1^i l^i$, we reach (40).

APPENDIX B PROOF OF LEMMA 7

It suffices to show that equations (69)-(72) can be replaced with (75). The direct proof is straightforward as we reach to (75) by combining (71)-(72). Then, (69)-(70) can be omitted as they are irrelevant to cost function and other constraints. To prove the converse, using (75) we have

$$\sum_{i=1}^k (P_1^i + \tilde{P}_2^i) l^i \leq \sum_{i=1}^k (E_1^{i-1} + \tilde{E}_2^{i-1}), \quad k = 1, \dots, K+1, \quad (77)$$

Subtracting $\tilde{P}_2^i l^i \geq 0$ from both sides, results in

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=1}^k E_1^{i-1} + \Delta^i, \quad k = 1, \dots, K+1, \quad (78)$$

where $\Delta^i = \tilde{E}_2^{i-1} - \tilde{P}_2^i l^i, \forall i$. This yields

$$\sum_{i=1}^k \tilde{P}_2^i l^i = \sum_{i=1}^k \tilde{E}_2^{i-1} - \Delta^i, \quad k = 1, \dots, K+1, \quad (79)$$

which can be expressed as

$$\sum_{i=1}^k \tilde{P}_2^i l^i \leq \sum_{i=1}^k \tilde{E}_2^{i-1} - \Delta^i, \quad k = 1, \dots, K+1, \quad (80)$$

Now, if $\Delta^i > 0$, we define $\Delta^i = \delta_1^i$; otherwise, for $\Delta^i < 0$, we define $\Delta^i = -\delta_2^i$. Therefore, we have (69)-(72). It is obvious that $\delta_1^i, \delta_2^i, \forall i$ are not unique. This completes the proof.

TABLE III

POWER ALLOCATION FOR S AND R IN NC-EH- \mathcal{RC} . THE RESULTS ARE PRESENTED FOR SCENARIO 4 OF TABLE I, IN WHICH DISJOINT OPTIMIZATION ALGORITHM IS OPTIMAL. POWER ALLOCATION IN ALGORITHM 4 IS ALSO INCLUDED TO BE COMPARED WITH THAT OF CVX-TOOL WITH TWO-WAY ET.

Scenario	Solution Methods	Power Allocation for S, \mathbf{P}_1 and \mathbf{L}_1				Power Allocation for R, \mathbf{P}_2 and \mathbf{L}_2							
Scenario 4	Algorithm 2	\mathbf{P}_1	=	[5.5	5]	\mathbf{P}_2	=	[4.8333	10]		
		\mathbf{L}_1	=	[6	1]	\mathbf{L}_2	=	[6	1]		
	Disjoint Optimization Algorithm	\mathbf{P}_1	=	[5.4286]	\mathbf{P}_2	=	[4.8333	10]		
		\mathbf{L}_1	=	[7]	\mathbf{L}_2	=	[6	1]		
	CVX-Tool	\mathbf{P}_1	=	[5.4288	5.4272]	\mathbf{P}_2	=	[4.6786	4.7186	4.9065	10.3926]	
		\mathbf{L}_1	=	[6	1]	\mathbf{L}_2	=	[2	2	2	1]	
	CVX-Tool with one-way ET	\mathbf{P}_1	=	[5.4286]	\mathbf{P}_2	=	[4.7325	4.7533	4.8809	10.2667]	
		\mathbf{L}_1	=	[7]	\mathbf{L}_2	=	[2	2	2	1]	
	Algorithm 4	\mathbf{P}_1	=	[6.2083	11.25]	\mathbf{P}_2	=	[4.6563	8.4375]	
		\mathbf{L}_1	=	[6	1]	\mathbf{L}_2	=	[6	1]	
	CVX-Tool with two-way ET	\mathbf{P}_1	=	[21.6191	21.5233	21.4378	38.4547]	\mathbf{P}_2	=	[0.8036	0.8275	0.8489	1.6363]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]

TABLE IV

POWER ALLOCATION FOR S AND R IN NC-EH- \mathcal{RC} . THE RESULTS ARE PRESENTED FOR SCENARIO 6 OF TABLE I, IN WHICH BOTH OF THE PROPOSED SUBOPTIMAL ALGORITHMS ARE OPTIMAL. POWER ALLOCATION IN ALGORITHM 4 IS ALSO INCLUDED TO BE COMPARED WITH THAT OF CVX-TOOL WITH TWO-WAY ET.

Scenario	Solution Methods	Power Allocation for S, \mathbf{P}_1 and \mathbf{L}_1					Power Allocation for R, \mathbf{P}_2 and \mathbf{L}_2						
Scenario 6	Algorithm 2	\mathbf{P}_1	=	[3.5	5.5	9]	\mathbf{P}_2	=	[4.25	5.5	12]
		\mathbf{L}_1	=	[2	4	1]	\mathbf{L}_2	=	[4	2	1]
	Disjoint Optimization Algorithm	\mathbf{P}_1	=	[3.5	5.5	9]	\mathbf{P}_2	=	[4.25	5.5	12]
		\mathbf{L}_1	=	[2	4	1]	\mathbf{L}_2	=	[4	2	1]
	CVX-Tool	\mathbf{P}_1	=	[3.5	5.4989	5.5011	9]	\mathbf{P}_2	=	[3.3394	4.8254	5.4913	12.6878]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]
	CVX-Tool with one-way ET	\mathbf{P}_1	=	[3.5	5.4998	5.5002	9]	\mathbf{P}_2	=	[3.3420	4.8907	5.4962	12.5423]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]
	Algorithm 4	\mathbf{P}_1	=	[5.3750	6.8750	14.25]	\mathbf{P}_2	=	[4.0313	5.1563	10.6875]
		\mathbf{L}_1	=	[4	2	1]	\mathbf{L}_2	=	[4	2	1]
	CVX-Tool with two-way ET	\mathbf{P}_1	=	[18.5759	18.5001	23.6885	48.8493]	\mathbf{P}_2	=	[0.7310	0.75	0.9529	2.0377]
		\mathbf{L}_1	=	[2	2	2	1]	\mathbf{L}_2	=	[2	2	2	1]

APPENDIX C

PROOF OF LEMMA 8

In optimal solution of Algorithm 4, we have $\tilde{C}_1 = \tilde{C}_2$. Therefore, the problem is

$$\begin{aligned} \max_{P_1, \tilde{P}_2, \delta_1, \delta_2} \sum_{i=1}^{K+1} \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) l^i \\ \text{s.t.} \quad (68) - (72). \end{aligned} \quad (81)$$

This is simplified to

$$\begin{aligned} \max_{P_1, \delta_1, \delta_2} \sum_{i=1}^{K+1} \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) l^i \\ \text{s.t.} \quad P_1^i \geq 0, \forall i, \text{ and } (69), (70), (71), \end{aligned} \quad (82)$$

where P_2^i is removed from the problem. Substituting $\delta_1 = \delta_1^*$ and $\delta_2 = \delta_2^*$, we have

$$\max_{P_1} \sum_{i=1}^{K+1} \mathcal{C} \left(\frac{[a^2]^\dagger P_1^i}{N_0} \right) l^i \quad (85)$$

$$\text{s.t.} \quad P_1^i \geq 0, \forall i, \quad (86)$$

$$\sum_{i=1}^k P_1^i l^i \leq \sum_{i=0}^{k-1} \mathfrak{E}_1^i, \forall k, \quad (87)$$

It is clear that the solution of this problem (convex optimization problem in single variable \mathbf{P}_1) is the shortest-path algorithm applied to modified EH pattern \mathfrak{E}_1 . Using a similar technique, we can achieve the desired result for \mathbf{P}_2 .

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